EXHIBIT B

4 ed

OPTICS

Eugene Hecht
Adelphi University

Addison Wesley

San Francisco Boston New York
Capetown Hong Kong London Madrid Mexico City
Montreal Munich Paris Singapore Sydney Tokyo Toronto

and

$$t_{\parallel} \equiv \left(\frac{E_{0t}}{E_{0i}}\right)_{\parallel} = \frac{2\dfrac{n_i}{\mu_i}\cos\theta_i}{\dfrac{n_i}{\mu_i}\cos\theta_t + \dfrac{n_t}{\mu_t}\cos\theta_i} \tag{4.39}$$

When both media forming the interface are dielectrics that are essentially "nonmagnetic" (p. 66), the amplitude coefficients become

$$r_{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} \tag{4.40}$$

and

$$t_{\parallel} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} \tag{4.41}$$

One further notational simplification can be made using Snell's Law, whereupon the Fresnel Equations for dielectric media become (Problem 4.39)

$$r_{\perp} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \tag{4.42}$$

$$r_{\parallel} = +\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \tag{4.43}$$

$$t_{\perp} = +\frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)} \tag{4.44}$$

$$t_{\parallel} = +\frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)} \tag{4.45}$$

A note of caution must be introduced here. Bear in mind that the directions (or more precisely, the phases) of the fields in Figs. 4.39 and 4.40 were selected rather arbitrarily. For example, in Fig. 4.39 we could have assumed that $\vec{\mathbf{E}}_r$ pointed inward, whereupon $\vec{\mathbf{B}}_r$ would have had to be reversed as well. Had we done that, the sign of $r_{\perp}$ would have turned out to be positive, leaving the other amplitude coefficients unchanged. The signs appearing in Eqs. (4.42) through (4.45), which are positive except for the first, correspond to the particular set of field directions selected. The minus sign in Eq. (4.42), as we will see, just means that we didn't guess correctly concerning $\vec{\mathbf{E}}_r$ in Fig. 4.39. Nonetheless, be aware that the literature is not standardized, and all possible sign variations have been labeled the *Fresnel Equations*. To avoid confusion *they must be related to the specific field directions from which they were derived.*

4.6.3 Interpretation of the Fresnel Equations

This section examines the physical implications of the Fresnel Equations. In particular, we are interested in determining the fractional amplitudes and flux densities that are reflected and refracted. In addition we shall be concerned with any possible phase shifts that might be incurred in the process.

Amplitude Coefficients

Let's briefly examine the form of the amplitude coefficients over the entire range of θ_i values. At nearly normal incidence ($\theta_i \approx 0$) the tangents in Eq. (4.43) are essentially equal to sines, in which case

$$[r_{\parallel}]_{\theta_i=0} = [-r_{\perp}]_{\theta_i=0} = \left[\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}\right]_{\theta_i=0}$$

We will come back to the physical significance of the minus sign presently. After expanding the sines and using Snell's Law, this expression becomes

$$[r_{\parallel}]_{\theta_i=0} = [-r_{\perp}]_{\theta_i=0} = \left[\frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t}\right]_{\theta_i=0} \tag{4.46}$$

which follows as well from Eqs. (4.34) and (4.40). In the limit, as θ_i goes to 0, $\cos\theta_i$ and $\cos\theta_t$ both approach one, and consequently

$$[r_{\parallel}]_{\theta_i=0} = [-r_{\perp}]_{\theta_i=0} = \frac{n_t - n_i}{n_t + n_i} \tag{4.47}$$

This equality of the reflection coefficients arises because the plane-of-incidence is no longer specified when $\theta_i = 0$. Thus, for example, at an air ($n_i = 1$) glass ($n_t = 1.5$) interface at nearly normal incidence, the amplitude reflection coefficients equal ± 0.2. (See Problem 4.45.)

When $n_t > n_i$ it follows from Snell's Law that $\theta_i > \theta_t$, and $r_{\perp}$ is negative for all values of θ_i (Fig. 4.41). In contrast, Eq. (4.43) tells us that $r_{\parallel}$ starts out positive at $\theta_i = 0$ and decreases gradually until it equals zero when $(\theta_i + \theta_t) = 90°$, since there $\tan \pi/2$ is infinite. The particular value of the incident angle for which this occurs is denoted by θ_p and referred to as the **polarization angle** (see Section 8.6.1). Notice that $r_{\parallel} \to 0$ at θ_p, just when the phase shifts 180°. That means we won't see the $\vec{\mathbf{E}}$-field do any flipping when θ_i approaches θ_p from either side. As θ_i increases beyond θ_p, $r_{\parallel}$ becomes progressively more negative, reaching -1.0 at 90°.

If you place a single sheet of glass, a microscope slide, on this page and look straight down into it ($\theta_i = 0$), the region

arriving on the surface over A. Similarly, $I_r A \cos\theta_r$ is the power in the reflected beam, and $I_t A \cos\theta_t$ is the power being transmitted through A. We define the **reflectance** R to be the ratio of the reflected power (or flux) to the incident power:

$$R \equiv \frac{I_r A \cos\theta_r}{I_i A \cos\theta_i} = \frac{I_r}{I_i} \tag{4.54}$$

In the same way, the **transmittance** T is defined as the ratio of the transmitted to the incident flux and is given by

$$T \equiv \frac{I_t \cos\theta_t}{I_i \cos\theta_i} \tag{4.55}$$

The quotient I_r/I_i equals $(v_r\epsilon_r E_{0r}^2/2)/(v_i\epsilon_i E_{0i}^2/2)$, and since the incident and reflected waves are in the same medium, $v_r = v_i$, $\epsilon_r = \epsilon_i$, and

$$R = \left(\frac{E_{0r}}{E_{0i}}\right)^2 = r^2 \tag{4.56}$$

In like fashion (assuming $\mu_i = \mu_t = \mu_0$),

$$T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i}\left(\frac{E_{0t}}{E_{0i}}\right)^2 = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right)t^2 \tag{4.57}$$

where use was made of the fact that $\mu_0\epsilon_t = 1/v_t^2$ and $\mu_0 v_t \epsilon_t = n_t/c$. Notice that at normal incidence, which is a situation of great practical interest, $\theta_t = \theta_i = 0$, and the transmittance [Eq. (4.55)], like the reflectance [Eq. (4.54)], is then simply the ratio of the appropriate irradiances. Since $R = r^2$, we need not worry about the sign of r in any particular formulation, and that makes reflectance a convenient notion. Observe that in Eq. (4.57) T is not simply equal to t^2, for two reasons. First, the ratio of the indices of refraction must be there, since the speeds at which energy is transported into and out of the interface are different, in other words, $I \propto v$, from Eq. (3.47). Second, the cross-sectional areas of the incident and refracted beams are different. The energy flow per unit area is affected accordingly, and that manifests itself in the presence of the ratio of the cosine terms.

Let's now write an expression representing the conservation of energy for the configuration depicted in Fig. 4.47. In other words, the total energy flowing into area A per unit time must equal the energy flowing outward from it per unit time:

$$I_i A \cos\theta_i = I_r A \cos\theta_r + I_t A \cos\theta_t \tag{4.58}$$

When both sides are multiplied by c, this expression becomes

$$n_i E_{0i}^2 \cos\theta_i = n_i E_{0r}^2 \cos\theta_i + n_t E_{0t}^2 \cos\theta_t$$

or

$$1 = \left(\frac{E_{0r}}{E_{0i}}\right)^2 + \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right)\left(\frac{E_{0t}}{E_{0i}}\right)^2 \tag{4.59}$$

But this is simply

$$R + T = 1 \tag{4.60}$$

where there was no absorption. It is convenient to use the component forms, that is,

$$R_\perp = r_\perp^2 \tag{4.61}$$

$$R_\parallel = r_\parallel^2 \tag{4.62}$$

$$T_\perp = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right)t_\perp^2 \tag{4.63}$$

and

$$T_\parallel = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right)t_\parallel^2 \tag{4.64}$$

which are illustrated in Fig. 4.48. Furthermore, it can be shown (Problem 4.71) that

$$R_\parallel + T_\parallel = 1 \tag{4.65}$$

and

$$R_\perp + T_\perp = 1 \tag{4.66}$$

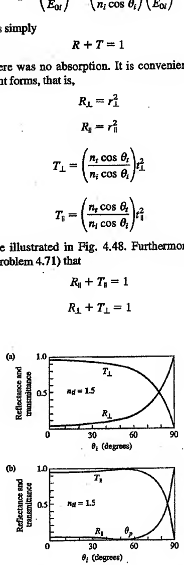
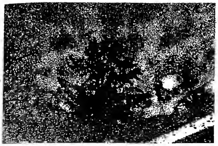


Figure 4.48 Reflectance and transmittance versus incident angle.



Looking down into a puddle (that's melting snow on the right) we see a reflection of the surrounding trees. At normal incidence water reflects about 2% of the light. As the viewing angle increases here its about 40° that percentage increases. (Photo by E.H.)

When $\theta_i = 0$, the incident plane becomes undefined, and any distinction between the parallel and perpendicular components of R and T vanishes. In this case Eqs. (4.61) through (4.64), along with (4.47) and (4.48), lead to

$$R = R_\parallel = R_\perp = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2 \tag{4.67}$$

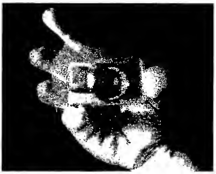
and

$$T = T_\parallel = T_\perp = \frac{4n_t n_i}{(n_t + n_i)^2} \tag{4.68}$$

Thus 4% of the light incident normally on an air–glass ($n_g = 1.5$) interface will be reflected back, whether internally, $n_i > n_t$, or externally, $n_i < n_t$ (Problem 4.72). This will be of concern to anyone who is working with a complicated lens system, which might have 10 or 20 such air–glass boundaries. Indeed, if you look perpendicularly into a stack of about 50 microscope slides (cover-glass sliders are much thinner and easier to handle in large quantities), most of the light will be reflected. The stack will look very much like a mirror (see photo). Roll up a thin sheet of clear plastic into a multiturned cylinder and it too will look like shiny metal. The many interfaces produce a large number of closely spaced *specular* reflections that send much of the light back into the incident medium, more or less, as if it had undergone a single frequency-independent reflection. A smooth gray-metal surface does pretty much the same thing —it has a large, frequency-independent specular reflectance—and looks shiny (that's what "shiny" is). If the reflection is diffuse, the surface will appear gray or even white if the reflectance is large enough.

Figure 4.49 is a plot of the reflectance at a single interface, assuming normal incidence for various transmitting media in air. Figure 4.50 depicts the corresponding dependence of the transmittance at normal incidence on the number of interfaces and the index of the medium. Of course, this is why you can't see through a roll of "clear" smooth-surfaced plastic tape, and it's also why the many elements in a periscope must be coated with antireflection films (Section 9.9.2).

At near normal incidence about 4% of the light is reflected back off each air-glass interface. Here because it's a lot brighter outside than inside the building, you have no trouble seeing the photographer. (Photo by E.H.)



Near normal reflection off a stack of microscope slides. You can see the image of the camera that took the picture. (Photo by E.H.)

of-incidence, we defined the *amplitude reflection coefficient* as $r_\parallel \equiv [E_{0r}/E_{0i}]_\parallel$, that is, the ratio of the reflected to incident electric-field *amplitudes*. Similarly, when the electric field is normal to the incident plane, we have $r_\perp \equiv [E_{0r}/E_{0i}]_\perp$. The corresponding irradiance ratio (the incident and reflected beams have the same cross-sectional area) is known as the *reflectance*, and since irradiance is proportional to the square of the amplitude of the field,

$$R_\parallel = r_\parallel^2 = [E_{0r}/E_{0i}]_\parallel^2 \quad \text{and} \quad R_\perp = r_\perp^2 = [E_{0r}/E_{0i}]_\perp^2$$

Squaring the appropriate Fresnel Equations yields

$$R_\parallel = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} \tag{8.26}$$

and

$$R_\perp = \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)} \tag{8.27}$$

Whereas $R_\perp$ can never be zero, $R_\parallel$ is indeed zero when the denominator is infinite, that is, when $\theta_i + \theta_t = 90°$. The reflectance, for linear light with $\vec{\mathbf{E}}$ parallel to the plane-of-incidence, thereupon vanishes; $E_{r\parallel} = 0$ and the beam is completely transmitted. This is the essence of Brewster's Law.

If the incoming light is unpolarized, we can represent it by two now familiar orthogonal, incoherent, equal-amplitude $\mathscr{P}$-states. Incidentally, the fact that they are equal in amplitude means that the amount of energy in one of these two polarization states is the same as that in the other (i.e., $I_{i\parallel} = I_{i\perp} = I_i/2$), which is quite reasonable. Thus

$$I_{r\parallel} = I_{r\parallel} I_i / 2I_{i\parallel} = R_\parallel I_i/2$$

and in the same way $I_{r\perp} = R_\perp I_i/2$. The reflectance in natural light, $R = I_r/I_i$, is therefore given by

$$R = \frac{I_{r\parallel} + I_{r\perp}}{I_i} = \tfrac{1}{2}(R_\parallel + R_\perp) \tag{8.28}$$

Figure 8.35 is a plot of Eqs. (8.26), (8.27), and (8.28) for the particular case when $n_i = 1$ and $n_t = 1.5$. The middle curve, which corresponds to incident natural light, shows that only about 7.5% of the incoming light is reflected when $\theta_i = \theta_p$. The transmitted light is then evidently partially polarized. When $\theta_i \neq \theta_p$ both the transmitted and reflected waves are partially polarized.

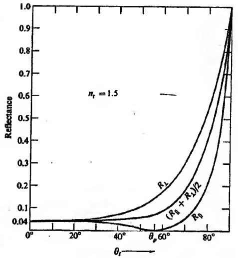


Figure 8.35 Reflectance versus incident angle.

It is often desirable to make use of the concept of the **degree of polarization** V, defined as

$$V = \frac{I_p}{I_p + I_n} \tag{8.29}$$

in which I_p and I_n are the constituent flux densities of polarized and "unpolarized" or natural light. For example, if $I_p = 4$ W/m^2 and $I_n = 6$ W/m^2, then $V = 40\%$ and the beam is partially polarized. With "unpolarized" light $I_p = 0$ and obviously $V = 0$, whereas at the opposite extreme, if $I_n = 0$, $V = 1$ and the light is completely polarized; thus $0 \leq V \leq 1$. One frequently deals with partially polarized, linear, quasimonochromatic light. In that case, if we rotate an analyzer in the beam, there will be an orientation at which the transmitted irradiance is maximum (I_{max}), and perpendicular to this, a direction where it is minimum (I_{min}). Clearly $I_p = I_{max} - I_{min}$, and so

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \tag{8.30}$$

Note that V is actually a property of the beam, which may be partially or even completely polarized before encountering any sort of polarizer.

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Project Manager: *Nancy Gee Benton*
Manufacturing Supervisor: *Vivian McDougal*
Cover Designer: *Blakeley Kim*
Production Service: HRS Interactive
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Photo Research: *Carolyn Eisen Hecht*

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